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Non-commutative gauge theory on D-branes in Melvin Universes

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Abstract

Non-commutative gauge theory with a non-constant non-commutativity parameter can be formulated as a decoupling limit of open strings ending on D3-branes wrapping a Melvin universe. We construct the action explicitly and discuss various physical features of this theory. The decoupled field theory is not supersymmetric. Nonetheless, the Coulomb branch appears to remain flat at least in the large N and large 't Hooft coupling limit. We also find the analogue of Prasad-Sommerfield monopoles whose size scales with the non-commutativity parameter and is therefore position dependent.

1 Introduction

It is generally believed that a proper understanding of quantum gravity requires taking quantum field theory beyond a framework based on locality. Non-commutative geometry is an important conceptual laboratory for exploring precisely this issue.

Gauge theories on non-commutative spaces arise naturally as a certain decoupling limit of open string dynamics in the presence of a background NSNS B-field. In recent years, there has been significant progress in our understanding of non-commutative field theories on flat spaces with constant non-commutativity. These theories arise in cases where the B-field is uniform [1]. Less is known about the generalization to the case where the non-commutativity parameter is non-constant. See e.g. [2–13]. Nonetheless, several concrete examples of non-commutative gauge theories with non-constant non-commutativities have been shown to arise as a decoupling limit of open strings in specific backgrounds with nontrivial background NSNS B-field [14–17].

A large class of backgrounds that can give rise to non-commutative gauge theories in the decoupling limit can be constructed by acting on flat space with twists and dualities. A particularly simple example of this construction is the Melvin universe [18] supported by the field strength of the NSNS B-field [19]. The decoupling limit of open string dynamics in this background was considered in [17]. It is a non-commutative gauge theory with the matter content of $\mathcal{N} = 4$ supersymmetry and a non-commutativity parameter that is position dependent. Such a theory can be formulated in terms of an action where the fields are multiplied using the $*$-product of Kontsevich [20]. The goal of this paper is to explore the physical features of this model in some detail.

The structure of supersymmetry is intricate for this theory. It is well known that type II string theory in Melvin universes generically breaks all of the supersymmetries [21–23]; the spectrum of open string fluctuations respects the Bose-Fermi degeneracy [24] but the supersymmetries are broken by the interactions. An easy way to see that this theory is not supersymmetric is to note that translation invariance is broken due to the non-constant non-commutativity parameter. This is incompatible with the supersymmetry algebra. However, we also note that supersymmetry is nonetheless restored in the infrared where the effects of non-commutativity becomes irrelevant.

In the large N and large 't Hooft coupling limit, it is straight forward to compute the DBI action of a probe D3-brane which measures the possible lifting of the Coulomb branch due to quantum effects [25]. It turns out however that no potential is generated along the Coulomb branch in the regime where the supergravity analysis is reliable. This is somewhat surprising in light of the fact that supersymmetry is clearly broken. It appears that the

effects of supersymmetry breaking are very mild for this theory.

Since the Coulomb branch is flat, this gauge theory can support Prasad-Sommerfield monopoles [26] in its broken phase. These monopoles have sizes proportional to the value of the non-commutativity parameter [27, 28] and as a result their size is position dependent in our model. They therefore provide a convenient probe for measuring the local value of the non-commutativity parameter.

This paper is organized as follows. In section 2, we review the construction of the Melvin universe in the presence of D3-branes as a background of type IIB string theory and describe the decoupling limit which gives rise to a non-commutative gauge theory with non-constant non-commutativity. In section 3, we present an explicit expression for the action for this gauge theory and analyze it's supersymmetry. In section 4, we construct the dual supergravity description for this theory and study the killing symmetry of that background. We also describe the Coulomb branch and the Prasad-Sommerfield monopoles of this model. We conclude in section 5.

2 IIB Melvin Universes and Non-commutative Gauge Theory

The Melvin universe is a non-asymptotically flat solution to the type IIB supergravity equations of motion. It has topology $R^3 \times R^1 \times R^6$ and is supported by the flux of a space-time dependent NSNS B-field. This background can be constructed by acting on flat space with a chain of twists and T-dualities. Upon wrapping D3-branes around this geometry and taking a specific decoupling limit, one obtains a non-commutative gauge theory with a space-time dependent non-commutativity parameter whose magnitude is proportional to the twist parameter. In this section we review the construction of the Melvin Universe and the decoupling limit.

The Melvin Universe can be obtained by applying the following chain of operations to flat space.

1. Start with a flat background in type IIB supergravity

$$ds^2 = -dt^2 + dr^2 + r^2 d\varphi^2 + dz^2 + \sum_{i=1,6} dy_i^2 \tag{2.1}$$

   where z is compactified on a circle with radius R.

2. T-dualize along z to obtain a background of type IIA supergravity

$$ds^2 = -dt^2 + dr^2 + r^2 d\varphi^2 + d\tilde{z}^2 + \sum_{i=1,6} dy_i^2 \tag{2.2}$$

where the radius of $\tilde{z}$ coordinate is $\tilde{R} = \alpha'/R$.

3. This geometry admits an isometry generated by a vector $\partial/\partial\varphi$. Given such an isometry vector, one can "twist" the compactification. By this, one means changing the Killing vector associated with the compactification from $(\partial/\partial\tilde{z})$ to $(\partial/\partial\tilde{z} + \eta\partial/\partial\varphi)$. Alternatively, one can think of the twist as first replacing

$$d\varphi \to d\varphi + \eta\, d\tilde{z} \tag{2.3}$$

so that the metric reads

$$ds^2 = -dt^2 + dr^2 + r^2(d\varphi + \eta d\tilde{z})^2 + d\tilde{z}^2 + \sum_{i=1,6} dy_i^2 \tag{2.4}$$

and then treating $\tilde{z}$ as the periodic variable of radius $\tilde{R}$ with φ fixed.

4. T-dualize along $\tilde{z}$ to obtain a solution of type IIB supergravity

$$\begin{aligned} ds^2 &= -dt^2 + dr^2 + \frac{r^2}{1+\eta^2 r^2} d\varphi^2 + \frac{1}{1+\eta^2 r^2} dz^2 + \sum_{i=1}^{6} dy_i^2 \\ B &= \frac{\eta r^2}{1+\eta^2 r^2} d\varphi \wedge dz \\ e^{\phi} &= g_s \sqrt{\frac{1}{1+\eta^2 r^2}}\ . \end{aligned} \tag{2.5}$$

This is a Melvin Universe supported by the flux of the NSNS B-field. It's global geometry is that of a teardrop.

In order to obtain a gauge theory, we consider D3-branes extended along the (t, r, φ, z) coordinates and apply the mapping of Seiberg and Witten [29]

$$(G + \frac{\theta}{2\pi\alpha'})^{\mu\nu} = [(g+B)_{\mu\nu}]^{-1}\ . \tag{2.6}$$

Applying this to the closed string background (2.5) we obtain the open string metric G and non-commutativity parameter θ

$$\begin{aligned} G_{\mu\nu} dx^\mu dx^\nu &= -dt^2 + dr^2 + r^2 d\varphi^2 + dz^2 \\ \theta^{\varphi z} &= 2\pi\alpha'\eta\ . \end{aligned} \tag{2.7}$$

The metric (2.7) is independent of α' and will therefore be the background for the decoupled theory. In order for the field theory on this background to have finite non-commutativity, we must take the particular scaling limit

$$\eta = \frac{\Delta}{\alpha'} \qquad \alpha' \to 0\ . \tag{2.8}$$

With this choice of scaling, the non-commutativity parameter has a finite limit

$$\theta^{\varphi z} = -\theta^{z\varphi} = 2\pi\Delta \ . \tag{2.9}$$

Further, it is clear in cartesian coordinates that the non-commutativity parameter is non-constant

$$\theta^{x_1 z} = -\theta^{z x_1} = -2\pi\Delta x_2, \qquad \theta^{x_2 z} = -\theta^{z x_2} = 2\pi\Delta x_1 \ . \tag{2.10}$$

It is divergence free

$$\partial_i \theta^{ij} = 0 \ . \tag{2.11}$$

and satisfies the Jacobi identity

$$\theta^{il}\partial_l\theta^{jk} + \theta^{jl}\partial_l\theta^{ki} + \theta^{kl}\partial_l\theta^{ij} = 0 \tag{2.12}$$

defining a Poisson structure. These features are useful in constructing a gauge invarant action on non-commutative spact-times [8, 11, 30].

3 Non-commutative gauge theory as a D-brane world volume theory

In this section, we write down the action for the non-commutative gauge theory we obtained in the decoupling limit of D3-branes embedded in the Melvin geometry (2.5). Because the non-commutativity tensor $\theta^{\mu\nu}$ we found in (2.10) is non-constant, we construct an associative product using the formula of Kontsevich [20]

$$\begin{aligned} f * g \ &= \ fg + i\frac{\theta^{\mu\nu}}{2}\partial_\mu f \partial_\nu g - \frac{1}{8}\theta^{\mu\nu}\theta^{\lambda\sigma}\partial_\mu\partial_\lambda f \partial_\nu\partial_\sigma g \\ &\quad -\frac{1}{12}\theta^{\mu\nu}\partial_\nu\theta^{\lambda\sigma}\left(\partial_\mu\partial_\lambda f\partial_\sigma g - \partial_\lambda f \partial_\mu\partial_\sigma g\right) + \mathcal{O}(\theta^3) \ . \end{aligned} \tag{3.1}$$

Taking θ constant in this formula gives the Moyal product. Naively, the action for non-commutative gauge theory is obtained by beginning with the ordinary gauge theory action and replacing all multiplications of the fields by the $*$-product.

This however is not suitable for constructing a gauge invariant action when the non-commutativity parameter is position dependent; differentiation does not respect the product rule

$$\partial_\mu(f * g) \neq \partial_\mu f * g + f * \partial_\mu g \ . \tag{3.2}$$

In order to properly formulate the action of non-commutative gauge theory, one can use the frame formalism introduced in [11]. This procedure will work for a generic Poisson bivector $\theta^{\mu\nu}$ on an arbitrary curved manifold. The non-commutative field theory we consider is

however extremely simple and one can write down an explicit form of the action in a rather straight forward manor. The result is in agreement with the general treatment of [11].

To do this, first recall from the previous section that the non-commutativity parameter takes on a simple constant form in polar coordinates (2.9). One can therefore define a Moyal-like product

$$f\#g = e^{\frac{i\theta^{\varphi z}}{2}(\partial_\varphi\partial_{z'}-\partial_{\varphi'}\partial_z)} f(t,r,\varphi,z)g(t,r,\varphi',z')\Big|_{\substack{\varphi=\varphi'\\ z=z'}} \ . \tag{3.3}$$

The #-product and $*$-product are not simply related by a change of coordinates, but are equivalent in the sense defined by Kontsevich [20]

$$R(f\#g) = R(f) * R(g) \ . \tag{3.4}$$

To order $\mathcal{O}(\theta^2) \sim \mathcal{O}(\Delta^2)$, $R(f)$ is given by

$$R(f) = f + \frac{4\pi^2\Delta^2}{24} r\partial_r\partial_z^2 R + \mathcal{O}(\Delta^3) \ . \tag{3.5}$$

Similar analysis of an explicit form of $R(f)$ can be found in [7].

In polar coordinates, it is natural to define the following set of unit norm vector fields

$$\partial_1 = \partial_t, \qquad \partial_2 = \partial_r, \qquad \partial_3 = \frac{1}{r}\partial_\varphi, \qquad \partial_4 = \partial_z \ . \tag{3.6}$$

They can also be written in terms of components

$$\partial_a = X_a^\mu \partial_\mu \ . \tag{3.7}$$

The vectors X_a define a natural local frame and one can therefore write the action of ordinary non-abelian gauge theory in the form

$$S = \frac{1}{2}\mathrm{tr}\int \sqrt{-G}\, G^{ab}G^{cd}\, F_{ac}F_{bd} \tag{3.8}$$

where $G_{ab} = g_{\mu\nu}X_a^\mu X_b^\nu = \eta_{ab}$ is the flat metric on the local frame, and

$$F_{ab} = \partial_a A_b - \partial_b A_a + ig[A_a, A_b]\,, \qquad A_a = X_a^\mu A_\mu \ . \tag{3.9}$$

In this formalism, it is straight forward to sprinkle the #'s to define a non-commutative theory

$$S = \frac{1}{2}\mathrm{tr}\int \sqrt{-G}\, G^{ab}G^{cd}\, F_{ac}\#F_{bd}, \qquad F_{ab} = \partial_a A_b - \partial_b A_a + igA_a\#A_b - igA_b\#A_a \tag{3.10}$$

since the ∂_a's respect the product rule

$$\partial_a(f\#g) = \partial_a f\#g + f\#\partial_a g \ . \tag{3.11}$$

However, some care is required in working with the action in this form. In particular, as integration by parts does not apply for one of the derivatives

$$\partial_3 = \partial_r \tag{3.12}$$

particular care is required when deriving the equations of motion.

Using these results and the automorphism $R(f)$, it is straight forward to formulate the action using the $*$-product. A derivation respecting the product rule can be defined according to

$$\delta_{X_a} f = R\partial_a R^{-1} f \tag{3.13}$$

since

$$\delta_{X_a}(f * g) = \delta_{X_a} R(R^{-1}f \# R^{-1}g) = \delta_{X_a} f * g + f * \delta_{X_a} g \ . \tag{3.14}$$

Then, the action

$$S = \frac{1}{2}\mathrm{tr}\int \sqrt{-G}\, G^{ab}G^{cd}\, F_{ac} * F_{bd}, \qquad F_{ab} = \delta_{X_a}A_b - \delta_{X_b}A_a + igA_a * A_b - igA_b * A_a \tag{3.15}$$

takes the form in which it was given in [11]. It is straight forward to generalize this construction to scalar and spinor fields and the non-commutative action for a field theory with $\mathcal{N} = 4$ matter content is given by

$$\begin{aligned} S &= \mathrm{tr}\int d^4x\sqrt{-G}\left[-\frac{1}{2}G^{ab}G^{cd}F_{ac} * F_{bd} - G^{ab}\sum_i D_a\Phi^i * D_b\Phi^i \right. \\ &\qquad \left. -i\bar{\psi} * \Gamma^\mu X^a_\mu D_a\psi + \frac{1}{2}\sum_{ij}[\Phi^i, \Phi^j]^2_* \ + \ldots\right] \end{aligned} \tag{3.16}$$

where

$$D_a\psi = \delta_{X_a}\psi - ig\,[A_a, \psi]_* \tag{3.17}$$

is the covariant derivative, and the ellipsis indicates the interaction terms involving the fermions.

In the remainder of this paper, we will explore various physical features of this theory. One issue which can be addressed immediately is that of supersymmetry. The matter content of this theory is consistent with $\mathcal{N} = 4$ supersymmetry. However, the fact that the non-commutativity parameter is not translation invariant implies that supersymmetry (which closes to translation) must be broken. This can be confirmed by explicit variation of the action. On the other hand, in the low energy limit where the effect of non-commutativity becomes irrelevant, all 32 supersymmetries of the $\mathcal{N} = 4$ theory are restored. Supersymmetry is also restored in the non-interacting limit $g^2_{YM} \rightarrow 0$, since non-commutativity only affects the interactions. This is consistent with the comment in footnote 36 of [24] concerning supersymmetry of the open string spectrum for the related embedding of D-branes in the Melvin universe.

4 Supergravity dual description of the non-commutative gauge theory

An effective approach in exploring the physical properties of a field theory is to study its supergravity dual. In this section, we review the supergravity dual and analyze three features of our model: supersymmetry, the Coulomb branch, and magnetic monopoles.

The supergravity background dual to the non-commutative gauge theory of interests was found in [17]. The background can be derived by beginning with the supergravity solution for a stack of D3-branes

$$\begin{aligned} ds^2 &= f(\rho)^{-1/2}(-dt^2 + dr^2 + r^2 d\varphi^2 + dz^2) + f(\rho)^{1/2}(d\rho^2 + \rho^2 d\Omega_5^2), \\ f(\rho) &= 1 + \frac{4\pi g N \alpha'^2}{\rho^4} \end{aligned} \tag{4.1}$$

and following the chain of dualities outlined in section 2. Then perform the scaling

$$\rho = \alpha' U, \qquad \eta = \frac{\Delta}{\alpha'} \tag{4.2}$$

while sending $\alpha' \to 0$ and keeping U and Δ fixed. The resulting geometry in the string frame is

$$\begin{aligned} ds^2 &= \alpha' \left(\frac{U^2}{\sqrt{\lambda}} \left[-dt^2 + dr^2 + \frac{r^2 d\varphi^2 + dz^2}{1 + \frac{\Delta^2 r^2 U^4}{\lambda}} \right] + \frac{\sqrt{\lambda}}{U^2} \left[dU^2 + U^2 d\Omega_5^2 \right] \right) \\ B_{\varphi z} &= \alpha' \frac{\Delta U^4 r^2}{\lambda + \Delta^2 U^4 r^2} \qquad\qquad C_{tr} = \alpha' \frac{2\pi}{g_{YM}^2} \frac{\Delta U^4 r}{\lambda} \\ C_{tr\varphi z} &= \alpha'^2 \frac{2\pi}{g_{YM}^2} \frac{U^4 r}{\lambda + \Delta^2 U^4 r^2} \qquad\qquad e^{\phi} = \frac{g_{YM}^2}{2\pi} \sqrt{\frac{\lambda}{\lambda + \Delta^2 U^4 r^2}} \end{aligned} \tag{4.3}$$

where $\lambda = 2 g_{YM}^2 N$.

Supersymmetry for this solution can be analyzed along the lines of [31]. In particular, we obtain the killing spinor equations for the Melvin Twisted D3-brane background and then consider the near horizon limit (4.2). Using a set of canonical vielbeins

$$\begin{aligned} E_t^0 &= E_r^1 = f^{-1/4} \qquad \frac{E_\varphi^2}{r} = E_z^3 = \frac{f^{1/4}}{\sqrt{1+\eta^2 r^2}} \qquad E_\rho^4 = f^{1/4} \\ E_{\varphi_5}^5 &= \rho f^{1/4} \qquad E_{\varphi_6}^6 = \rho f^{1/4} \sin\varphi_5 \quad \ldots. \quad E_{\varphi_9}^9 = \rho f^{1/4} \prod_{j=5..8} \sin\varphi_j \end{aligned} \tag{4.4}$$

with all other components vanishing, it is straight forward to obtain the dilatino variation

$$-\frac{1}{2} e^{-\frac{1}{4}\phi} \hat{\Gamma}^\mu \epsilon^* \partial_\mu \phi + \frac{1}{2} e^{\phi} \hat{\Gamma}^{\mu\nu\rho} \epsilon \mathcal{G}_{\mu\nu\rho} \ . \tag{4.5}$$

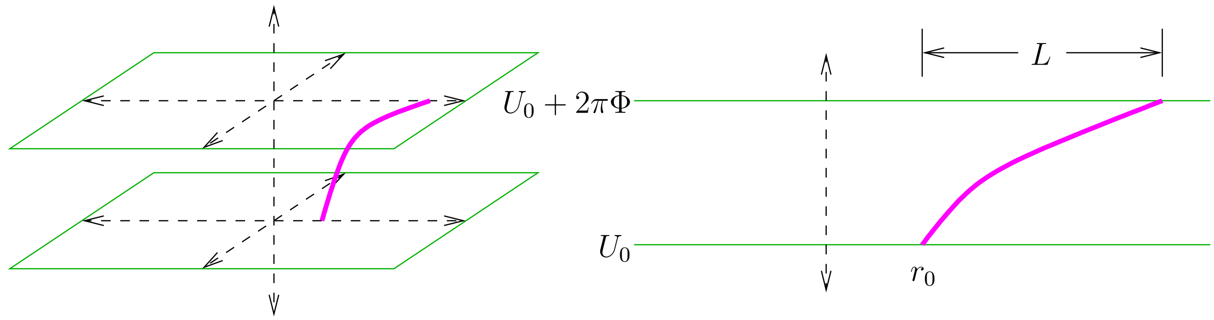

Figure 1: D-brane construction of the position dependent monopole

Here $\hat{\Gamma}^\rho = E_a^\rho \Gamma^a$ with the Γ^a being the standard 32×32 gamma matrices of Minkowski space-time in 10 dimensions, and

$$\mathcal{G}_{\mu\nu\rho} = i\left[3\partial_{[\mu}C_{\nu\rho]} + 3ie^{-\phi}\partial_{[\mu}B_{\nu\rho]}\right] \ . \tag{4.6}$$

The corresponding killing spinor equations are

$$\eta\left(\Gamma^{123}\epsilon + \eta r \Gamma^1 \epsilon^*\right) = 0 \tag{4.7}$$

where ϵ is a chiral spinor satisfying $\Gamma^{10}\epsilon = -\epsilon$ where $\Gamma^{10} = \Gamma^0\Gamma^1 \ldots \Gamma^9$. There is no non-tirvial solution to (4.7) either before or after taking the near horizon limit. This shows that as expected from the field theory analysis, supersymmetry is broken in the supergravity dual.

A simple physical observable of the field theory which can be computed from the supergravity dual is the potential along the Coulomb branch. Classically, these field theories have flat directions along field directions for which $[\Phi^i, \Phi^j] = 0$. For the $\mathcal{N} = 4$ SYM theory at strong coupling, the potential along the classical flat direction was computed using the DBI action of a probe D3-brane in $AdS_5 \times S_5$ [25]. It is straight forward to repeat this analysis for our model. Consider the DBI action for a probe D3-brane localized in U and extended along (t, r, φ, z) of the Melvin-Twisted D3-brane background

$$S = -\frac{1}{8\pi^3\alpha'^2}\int_M d^4x\, e^{-\phi} \det{}^{1/2}[G_{ab} + B_{ab}] + \frac{1}{8\pi^3\alpha'^2}\int_M \left(C^{(4)} + C^{(2)} \wedge B\right) \ . \tag{4.8}$$

When the background (4.3) is substituted into this action, one finds

$$S = -\frac{1}{8\pi^3\alpha'^2}\left[\int d^4x\, \frac{U^4}{\sqrt{\lambda}} - \int d^4x\, \frac{U^4}{\sqrt{\lambda}}\right] \ . \tag{4.9}$$

The action on this background is therefore Δ-independent and the NSNS and RR contributions cancel identically just as in the $\Delta = 0$ case [25]. This implies that the Coulomb branch remains flat for the non-commutative theory at large 't Hooft coupling, even when $U^4 \gg \frac{\lambda}{\Delta^2 r^2}$

where the deformation of the supergravity background (4.3) from $AdS_5 \times S_5$ is large. This is somewhat surprising as one would expect the moduli space to receive corrections. One possibility is that the moduli space being a zero momentum observable is insensitive to the non-commutativity. Nevertheless, it is surprising that there does not seem to be contributions coming from loop corrections. In regard to the potential along the Coulomb branch, it therefore appears that effect of supersymmetry breaking is mild.

The fact that the Coulomb branch is flat implies that one can turn on vacuum expectation values for the adjoint scalars and support a Prasad-Sommerfield monopole [26]. Simple physical properties of these monopoles can then be studied quite effectively using the dual supergravity formalism [27]. In this spirit, consider a probe D-string suspended between a pair of D3-branes in the background (4.3). Let the the pair of D3-branes be located at $U = U_0$ and $U = U_0 + 2\pi\Phi$. The DBI action for the probe D-string is given by

$$S = -\frac{1}{2\pi\alpha'}\int_{U_0}^{U_0+2\pi\Phi} dU \left[e^{-\phi}\sqrt{-G_{tt}\left(G_{UU} + r'(U)^2 G_{rr}\right)} - C_{0r} r'(U)\right] \tag{4.10}$$

where we have parameterized the static configuration of the string by a function $r = r(U)$. When this ansatz and the background (4.3) are substituted into (4.10) the action becomes

$$S = -\frac{1}{g_{YM}^2\lambda}\int_{U_0}^{U_0+2\pi\Phi} dU \left[\sqrt{\left(\lambda + \Delta^2 U^4 r(U)^2\right)\left(\lambda + U^4 r'(U)^2\right)} - \Delta U^4 r(U) r'(U)\right] . \tag{4.11}$$

The boundary conditions on the D-string imply that

$$\frac{dr}{dU} = \Delta r \tag{4.12}$$

at the boundaries $U = U_0$ and $U = U_0 + 2\pi\Phi$. The solution to (4.12)

$$U = U_0 + \frac{1}{\Delta}\ln\frac{r}{r_0} \tag{4.13}$$

in fact, also extremizes (4.11) for all values U and describes a static configuration of the D-string. Here r_0 is the position of the endpoint of the D-string along r when $U = U_0$. This solution is position dependent and corresponds to a D-string stretched between D3-branes extending along Melvin geometry.

In the case of constant non-commutativity, it was found that the magnetic monopole becomes a dipole with a finite length proportional to the non-commutativity parameter Δ and the expectation value of the adjoint Higgs field Φ [27, 28]

$$L = 2\pi\Delta^2\Phi \ . \tag{4.14}$$

In the Melvin-Twist gauge theory where the non-commutativity parameter is non-constant, we find that the length of the dipole is position dependent. Explicitly

$$L = r_0\left(e^{2\pi\Delta\Phi} - 1\right) \ . \tag{4.15}$$

Therefore the length of the dipole is proportional to the local magnitude of the non-commutativity parameter. For small values of $\Delta\Phi$, the relation between the non-commutativity parameter and the length of the dipole is exactly the same as in the case of constant non-commutativity. For arbitrary values of $\Delta\Phi$, the magnetic monopoles acquire length in the radial direction. This suggests that the S-dual of this theory, where these monopoles become the fundamental degrees of freedom, is an NCOS [32–34] with non-vanishing commutation relations between the t and r coordinates.[1]

It is also straightforward to calculate the mass of the monopole from the DBI action. We find

$$M = \frac{2\pi\Phi}{g_{YM}^2} . \tag{4.16}$$

This mass is identical to the ordinary SYM monopole. Interestingly, this indicates that although the length of the monopole is position dependent the mass is not and agrees with the constant non-commutativity case. It is also interesting that even though supersymmetry is broken, these monopoles mimic BPS monopoles and can therefore be moved freely through out the D3-brane world volume without feeling any force. These monopoles therefore serve as a useful physical probe to measure the position dependence of the non-commutativity parameter.

5 Conclusion

The purpose of this article was to explore physical aspects of a non-commutative gauge theory with non-constant non-commutativity. As a particular example, we focused on the Melvin-Twist non-commutative gauge theory which can be viewed as a special case of the gauge theory whose action was constructed in [11]. What makes the model considered in this paper special is that it arises as the decoupling limit of open strings ending on D3-branes extended along the Melvin universe supported by the flux of an NSNS B-field [17]. As such, we were able to utilize techniques in string theory to extract physical information from this theory. Specifically, we studied the supersymmetry, the Coulomb branch, and the Prasad-Sommerfield magnetic monopoles, using the dual supergravity formulation.

The matter content of the model under consideration is that of $\mathcal{N} = 4$ SYM theory in four dimensions. Although the free theory is supersymmetric, interactions induced by the non-commutativity necessarily break supersymmetry as translation invariance is broken by the non-commutativity parameter. Interestingly, although supersymmetry is broken in this model, we found that no potential is generated along the Coulomb branch through

[1] Supergravity dual of NCOS with time dependent non-commutativity parameter which arises as the S-dual of [15] is described in [35].

quantum corrections in the large N and large 't Hooft coupling limit. In light of this, we constructed classical configurations of D-strings stretched between D3-branes representing Prasad-Sommerfield monopoles. Using the brane configuration, we uncovered the interesting fact that although the mass of the monopoles are the same as in commutative SYM theory, the length is position dependent. This reflects the fact that the non-commutativity parameter is non-constant and is in line with the findings of [27] for the constant non-commutativity case. In contrast with the constant non-commutativity case, our model is non-supersymmetric and it is quite remarkable that the monpoles of this theory mimic BPS monopoles.

The conclusions regarding the monopoles were based entirely on the dual supergravity formalism. However, since the action of the field theory under study is known, it should be possible to repeat the analysis of the Coulomb branch potential perturbatively [9, 10] and also construct exact solutions using the Nahm construction along the lines of [28]. It would be interesting to see if such analysis reproduces the basic properties of the monpoles uncovered using the dual supergravity description. One could further consider issues such as deformations of the moduli-space metric due to the non-commutativity and it's effect on monopole scattering [36].

Lastly, let us comment that the brane probe analysis of the supergravity dual carried out in this paper can be repeated for other supergravity duals of non-commutative gauge theories with non-constant non-commutativity [10, 14, 16, 17]. Some of these models have a time-dependent non-commutativity parameter and may give rise to interesting new physics.

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